The performance of bank portfolio optimization

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Abstract

Given a liability structure, the bank portfolio optimization determines an asset allocation that maximizes profit, subject to restrictions on Basel III ratios and credit, liquidity, and market risks. Bank allocation models have not been tested using historical data. Using an optimization model based on turnover constraints, we develop such

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tests, which document the superior performance of optimization strategies compared to heuristic rules, resulting in an average annual out-of-sample outperformance of 15.1% in terms of Return on Equity using our data set. This outperformance is remarkable and contrasts with the reported underperformance of several portfolio optimization methods in the case of investment management.

1 Introduction

The optimization of bank balance sheets consists of the choice of the allocations in the different asset classes. It involves several variables, namely the prospective returns on asset classes, the regulatory framework, the accounting rules, and internal risk estimates.

To the best of our knowledge, there have been no historical tests of bank portfolio optimization methodologies, unlike, for instance, on investment portfolio optimization [25, 27, 33, 36], where numerous papers have addressed the historical performance of different optimization methodologies. As many articles have shown in the case of investment portfolios (see for instance [9, 11]), optimization methods guarantee best returns *ex-ante* (or in-sample), but often do not outperform heuristic strategies *ex-post* (out-of-sample). Due to the underperformance of many investment portfolio optimization approaches, Bridgewater Associates has advocated using All Weather Portfolios [7], related to Risk Parity Portfolios [2, 9, 34, 35], which are successful heuristics.

Bank portfolio management, although more involved than investment portfolio management due to the several risks to be considered, has a significant advantage in that the returns of the different asset classes are more predictable. The mark-to-market accounting associated with investment management portfolios makes predicting returns very difficult since market prices account for most of the returns, and these are very difficult to predict. In contrast, the prediction of returns for asset classes in bank portfolio optimization is easier: the returns for loans and securities at amortized cost, which often are the bulk of banks' assets, are essentially the interest rate (which is known for fixed-rate assets), and the expected loss, which is much easier to forecast than variations in market prices. This observation propelled us to investigate the out-of-sample performance of bank optimization methods.

Bank portfolio optimization falls into the realm of asset-liability management. Bank balance sheet optimization models have been available since the eighties. We cite a few references. Kusy and Ziemba [29] have created a framework for calculating optimal balance sheets using the stochastic nature of cash outflows. Kosmidou and Zopounidis [28] have devised a simulation-optimization framework considering the balance sheet's interest rate risk. The regulatory and accounting frameworks have evolved considerably since then: for instance, Basel III has been implemented, and regulators actively monitor capital and liquidity ratios; credit risk measurement has also grown significantly since then.

A few papers have recently addressed bank asset structure optimization. Halaj [23, 24] has devised methodologies for calculating the optimal asset structure of a bank in the presence of solvency and liquidity restrictions. Schmaltz *et al.* [37] have also researched optimal balance sheets in the presence of regulatory constraints. Uryasev, Theiler, and Serraino [41] conducted optimization using different aggregation measures, while Sirignano, Tsoukalas, and Giesecke [38] ran large-scale asset allocation. Yan, Zhang, and Wang [42] devised a robust model for asset allocation, while Brito and Júdice [8] tackled the allocation problem under IFRS 9. Júdice and Zhu [26] used linear programming duality to solve the asset-liability management problem.

To the best of our knowledge, none of the models for bank portfolio optimization have been tested out-of-sample. Out-of-sample tests are critical to evaluate the model robustness and its practical use. As we mentioned, the bulk of investment portfolio optimization models are discarded in practice due to their poor performance out-of-sample. As our research will show, this is not the case for a properly calibrated bank portfolio optimization model, which we will present.

With this literature review in mind, our research provides the following contributions:

1. First, we devise an optimal bank asset allocation model, given a liability structure, using global turnover constraints, which are easy to calibrate, and have been used in the context of portfolio optimization [10]. Turnover constraints restrict variations on the asset proportions and thus prevent large fluctuations in allocations each year. This sort of constraints stem from the fact that banks cannot change their asset composition very quickly, since loans are hard to liquidate in a short-time period. Consequently, a major change in the balance sheet is only achieved if market conditions show a steady trend over time. We focus solely on the asset structure, given that it is easier to change the asset structure than the liability structure. For example, as documented in [22], growing the deposit base is often difficult as retail deposits tend to be sticky. Also, equity capital may be difficult to obtain particularly at times of increased financial stress.

2. We use extensive historical data to devise a testing framework of optimization and heuristic strategies, addressing both the performance and the stability of the allocations.

3. We document the excessive sensitivity of optimization methodologies without global turnover constraints. The allocations without turnover constraints can vary in our setting up to about 50% in a year, which is infeasible in practice. For example, a bank cannot change the allocation of its consumer credit portfolio from 10% to 60% in a year, unless it makes an acquisition of a large consumer credit business unit, which may not be readily available. We demonstrate that turnover methodologies yield smoother allocation trajectories, which enable them to be used in practice.

4. Finally, we document the superiority of optimization strategies when compared to classical heuristic strategies, resulting in an average out-of-sample outperformance of 1.51% in return on assets and of 15.1% in return on equity.

These contributions show that bank asset optimization methodologies are suitable to be used in practice, since they can combine the superior profitability presented by the optimization models with allocations that can be implemented by bank management.

This paper is organized as follows. Section 2 describes the model and the return and risk parameters used as inputs. Section 3 conducts an extensive analysis of out-of-sample results on the performance and stability of the optimization method with turnover constraints against a series of heuristic benchmarks that we adapt to the banking context. Section 4 concludes the manuscript.

2 Model description

In this section, we develop the model for the optimal asset structure. We assume that the liability structure is fixed, for the reasons explained in the introduction. Section 2.1 describes each equation of the model and Section 2.2 gives the rationale for the estimations of the internal parameters.

Throughout the paper, we assume the following time notation. At the beginning of year t, the bank decides its optimal allocation (x_t) based on the information it has up to the end of the previous year $t-1$ (*ex-ante* evaluation), namely interest rates and default rates (r_{t-1} and PD_{t-1}). By the end of year t, the bank will know if the model performed well during that year based on the changes in asset prices and default rates observed at the end of year t (*ex-post* evaluation). In this section, we address the optimal asset allocation model. The performance of the model is evaluated in Section 3. We omit the indices t in this section for clarity of exposition, as it is only concerned with the model.

We intend to apply the algorithm regularly over the period under study following a rolling window methodology. Consequently, the algorithm starts with the portfolio x^0, which is the one currently used in the first year of the study. Next, it finds the optimal asset structure x^* for the next year, which satisfies the Basel III and turnover constraints, given a prediction of the rates, defaults and risk of each asset obtained from the previous ten year's data.

Asset	Description
A_1	Cash
A_2, A_3	Mortgage and Personal Loans, respectively
A_4, A_5	Treasury bonds AFS and HTM, respectively
A_6, A_7	Corporate bonds AFS and HTM, respectively

Table 1: Different segments where the bank can make its investments.

Then, x^0 is updated with x^* and the model is applied to obtain the optimal solution for the following year and so on.

2.1 The proposed model

Let us consider a bank that has to make the decision on how to allocate its assets. We assume that the bank's funding comes from deposits, money market funding, issued bonds, and shareholders' capital. The proportions that come from each of these funds are denoted by l_i, $i \in \{1, 2, 3, 4\}$, respectively, which are constants and inputs to the problem, for the reasons explained in the introduction. For example, $l_2 = 0.2$ means that 20% of the bank's funding comes from money market funds. This set of liabilities is representative of bank's funding, as documented for example in [6] and [22].

Let $A = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$ be the set of assets described in Table 1. This set describes a great part of the activity of many banks, and for all these aggregates we have historical data as we will see in the following sections. We note that equities are excluded, since typically equity investments are a tiny fraction of banks' portfolios. For example, the European Banking Authority [13] reports that equity investments represent 1.3% of European banks' total assets as of the end of 2022.

HTM designates *Held-To-Maturity* assets, while AFS designates *Available-For-Sale* assets. Let $A_L = \{A_2, A_3, A_5, A_7\}$ be the subset of assets associated with long holding periods, that is, loans and HTM assets. The distinction between AFS and HTM bonds is important. When a bank classifies a bond as available for sale (AFS), it signals the possibility of selling the bond in the market. If a bank classifies a bond as held-to-maturity (HTM), it is signaling that it will not sell the bond in the market. Using the accounting rules, AFS bond devaluations impact comprehensive income and the bank's return as a consequence. However, the accounting rules for HTM specify that bond devaluations have no impact on comprehensive income and the return on the bank.

Additionally, define $\Omega = \{x \in \mathbb{R}^7 : \sum_{i \in A} x_i = 1, x_i \geq 0\}$ as the set of admissible portfolios. In our model, we assume that the bank has to decide on a particular allocation $x \in \Omega$ at the beginning of a certain year, taking into account a previous decision $x^0 \in \Omega$ made in the beginning of the previous year.

Our model distinguishes between legacy ($\hat{x}_i$) and new ($\tilde{x}_i$) contracts for each asset. As a consequence, we also distinguish between the interest rate on legacy contracts, $\hat{r}_i$, and the interest rate on new ones, r_i. The amount of legacy contracts is obtained through repayments, so that $\hat{x}_i = (1 - \alpha_i)x_i^0$, where α_i is the repayment portion on asset i. Finally, legacy and new contracts fulfill the portfolio, so $x_i = \hat{x}_i + \tilde{x}_i$, $\forall i \in A$.

Since capital and liabilities are given as inputs to the problem, the calculations that depend on them are considered constants, as we will explain in detail below. We note that these constants depend upon the liability structure, and heavily influence the problem. Thus, the liability structure is an important input to the problem. Given the set of inputs, we propose the following model:

$$
\begin{aligned}
\max_{x\in\Omega} \quad & r(x) && (1)\\
\text{subject to} \quad & \frac{\sum_{i\in A}\lambda_i x_i}{\Lambda} \geq K_1, && (2)\\
& \frac{N}{\sum_{i\in A}\nu_i x_i} \geq K_2, && (3)\\
& \frac{C - IRR - \sqrt{V(x)}}{\sum_{i\in A} RW_i x_i} \geq K_3, && (4)\\
& \frac{\sum_{i\in A} S_i x_i}{M} \geq K_4, && (5)\\
& \hat{x}_i = (1-\alpha_i)x_i^0, \qquad \forall i \in A && (6)\\
& x_i = \hat{x}_i + \tilde{x}_i, \qquad \forall i \in A && (7)\\
& x_i - x_i^0 \leq y_i, \qquad x_i^0 - x_i \leq z_i, \qquad \forall i \in A && (8)\\
& \sum_{i\in A} (y_i + z_i) \leq h && (9)\\
& y_i \leq \alpha_i x_i^0, \qquad \forall i \in A_L && (10)\\
& z_i \leq \alpha_i x_i^0, \qquad \forall i \in A_L && (11)\\
& y_i,\, z_i \geq 0, \qquad \forall i \in A && (12)
\end{aligned}
$$

We now explain each equation, one by one.

First we look at the return. The objective function of our model (1) corresponds to the prospective return of the bank sheet, and it is necessary to differentiate prospective returns from legacy and new contracts:

$$
r(x) = \sum_{i\in A_L} \left(\hat{x}_i\hat{r}_i + \tilde{x}_i r_i - x_i LGD_i \overline{PD}_i\right) + \sum_{i\notin A_L} x_i r_i, \tag{13}
$$

where r_i is the interest rate on new contracts and $\hat{r}_i$ is the interest rate on legacy contracts. $\overline{PD}_i$ denotes the estimated probability of default, whereas LGD_i stands for the loss given default. r_i, $\hat{r}_i$, $\overline{PD}_i$ and LGD_i are all inputs to the model.

The equation (2) is a restriction on the Liquidity Coverage ratio, a key metric that was developed in the aftermath of the Subprime Crisis by the Basel Committee in Banking Supervision to ensure that banks had enough liquid assets to withstand liquidity outflows from creditors (such as deposit runs) [4]. This restriction posits that banks should have enough high quality liquid assets that can be sold in case there are massive withdrawals from costumers. The ratio compares the realizable value of assets $\sum_{i \in A} \lambda_i x_i$ with the potential withdrawals from liabilities $\sum_{j=1}^4 \bar{\lambda}_j l_j$, where $\lambda_i, \bar{\lambda_j} \in [0,1]$ are constants. The higher the liquidity of asset class i, the higher is the weight λ_i. For liabilities, the higher values of $\bar{\lambda}_j$ correspond to the liabilities that experience higher withdrawals in episodes of stress, such as money market funds. The ratio and the limit prescribed by the regulators is thus

$$\frac{\sum_{i \in A} \lambda_i x_i}{\sum_{j=1}^4 \bar{\lambda}_j l_j} \geq K_1. \tag{14}$$

By taking $\Lambda = \sum_{j=1}^4 \bar{\lambda}_j l_j$, which is constant, we get restriction (2).

On the left hand of the constraint (3) we have the regulatory ratio NSFR (Net Stable Funding Ratio) that evaluates if the bank has enough stable liabilities to fund illiquid assets. This ratio is given by

$$\frac{\sum_{j=1}^4 \bar{\nu}_j l_j}{\sum_{i \in A} \nu_i x_i}, \tag{15}$$

and compares the available stable funding $\sum_{j=1}^4 \bar{\nu}_j l_j$ with the required stable funding $\sum_{i \in A} \nu_i x_i$. On the one hand, the required stable funding is the amount of stable funds the bank should hold to finance illiquid assets. The weights for the required stable funding $\nu_i \in [0,1]$ reflect the illiquidity of assets, so that if i is more illiquid, it receives a greater weight. On the other hand, the available stable funding reflects the amount of stable financing that the regulator thinks the bank has. It is given by a weighted sum of the liabilities, where the weights $\bar{\nu}_j \in [0,1]$ reflect the stability of the funding. As a result, $\bar{\nu}_j$ will be higher for liabilities that are less likely to experience massive withdrawals, such as long-term bonds or retail deposits. By taking $N = \sum_{j=1}^4 \bar{\nu}_j l_j$ and enforcing a minimum limit K_2 on the NSFR, we get restriction (3).

On equation (4), the bank evaluates if its common equity tier 1 ratio is above a certain threshold K_3 even after a shock in interest rates and devaluations or defaults on assets. Let us recall that the common equity tier 1 ratio is given by the ratio of capital to risk-weighted assets. The latter are calculated using the formula

$$\sum_{i \in A} RW_i x_i, \tag{16}$$

where RW_i is a risk weight that specified by regulators that describes the riskiness of asset i. In the numerator of the formula, we include the capital that the bank has and include two shocks, which we describe below:

- A first shock is a net interest margin shock, which comes from the repricing of liabilities. If interest rates go up, the bank will lose $\sum_{j=1}^{4} \delta_j l_j$, where δ_j measures the decrease in net interest margin in liability j due to an increase in interest rates. Note that we assume that all the assets reprice at one year or more, so they will not influence the net interest margin if interest rates go up. Thus the net interest margin shock is a constant $IRR = \sum_{j=1}^{4} \delta_j l_j$ that depends upon the structure of the liabilities (l_1, l_2, l_3, l_4), which is an input to the problem.

- A second shock $V(x) = \sum_{i \in A} (\sigma_i x_i)^2$ is determined by devaluations on market assets and defaults on long-term assets, where σ_i is the risk associated with defaults or devaluations on asset class i. We assume that $\sigma_1 = \sigma_5 = 0$ since their credit risk is very low and the risk of market devaluations is negligible (HTM assets are not impacted by market devaluations). $V(x)$ is thus a risk function for the asset structure that depends on the individual risks for each asset class. The risk factor σ_i for loans and corporate bond HTM ($i \in \{2, 3, 7\}$) will be given by the credit VaR which is given by the difference between the unexpected loss at 99.9% and the expected loss [21], that is $\sigma_i = UL_i(0.999) - EL_i$ where $UL_i(0.999) = N\left(\sqrt{\frac{1}{1-\rho_i}} \times N^{-1}\left(\overline{PD}_i\right) + \sqrt{\frac{\rho_i}{1-\rho_i}} \times N^{-1}(0.999)\right) \times LGD_i$ and $EL_i =$

$\overline{PD}_i \times LGD_i, i \in \{2, 3, 7\}$. Finally, the risk factor for AFS bonds will be given by the Market VaR, which is $\sigma_i = N^{-1}(0.95)s_i, i \in \{4, 6\}$, where s_i is the standard deviation of the return of asset i.

Expression (5) is a coverage ratio that determines if market-related assets cover wholesale liabilities. Market-related assets are given by $\sum_{i \in A} S_i x_i$, where S_i is zero for loans, and 1 for the remaining assets, whereas wholesale liabilities are given by $M := l_2 + l_3$, i.e., the total proportion of money market funds and issued bonds in the liability structure.

The following constraint (6) describes the legacy assets that we have described prior to the formulation of the model, whereas (7) states that the proportions of legacy assets $\hat{x}_i$ and new assets $\tilde{x}_i$ in a certain asset class i have to be equal to the total proportion x_i in asset class i.

The global turnover restrictions are shown in equations (8) - (9), where we posit that the global variation of the assets (as measured by the sum of the absolute value of the differences to the previous portfolio x^0) is not greater than a constant h. This constant essentially describes the fact that banks should not reallocate their assets in an abrupt manner, given the difficulty in reallocating large proportions of assets in practice. Such difficulty stems from the practical constraints on selling loan portfolios and whole business units, or reallocating systems or people to other business units. Variables y_i and z_i allow us to write this constraint in a linear manner.

Finally, local turnover constraints are shown in equations (10) and (11). These equations specify that, for long-term assets in A_L, the bank cannot reallocate assets by an amount greater than $\alpha_i x_i^0$. This restriction stems, to a great extent, from the fact that banks cannot liquidate legacy assets easily, so we enforce that $x_i \geq \hat{x}_i$, which is equivalent to $x_i^0 - x_i \leq \alpha_i x_i^0$, and it is obtained from equations (8) and (10). We also enforce a local upper bound on x, given by $x_i - x_i^0 \leq \alpha_i x_i^0$, and in the tests this bound will be relaxed on some versions of the model.

Although the theme of limited reinvestments by repayments has also been addressed in [23], that research neither considers the upper bound local turnover constraint (10) nor the global turnover constraint (9). Consequently, that model allows an arbitrarily large investment in assets belonging to A_L which can not be easily divested in the following years. Additionally, the global turnover is not restricted, leading to solutions that show large variations which are hard to accomplish in practice.

This model represents a nonlinear problem since the constraint (4) is nonlinear. This constraint can be converted to

$$IRR + \sqrt{V(x)} + K_3 \sum_{i \in A} RW_i x_i - C \leq 0. \tag{17}$$

The left-hand side, as a sum of convex functions, is convex. Since the other constraints are linear (or can be transformed into linear ones), all constraints are convex, wherefore the admissible region is convex. Consequently, the optimization problem is convex, ensuring that the local optimum is global.

For that reason, we used the interior-point method (see for instance [30]) to find the optimal solution x^* for this model through the *fmincon* tool that belongs to the Matlab Optimization Toolbox.

2.2 Model parameter estimation

As we already mentioned, we assume that the liability structure is fixed and corresponds to:

	Liabilities			
	Deposits	Money Market	Issued Bonds	Capital
Allocation	0.5	0.2	0.2	0.1

Table 2 reports the fixed input data to our model. In this table, IRR represents the sensitivity of net interest income to a 300 basis point shock in interest rates. Since all the asset classes are at fixed rates, the liability structure determines completely this sensitivity. In our test case, a 300 basis point increase in rates has a negative impact of 1.1% in the balance sheet.

We now address the estimation of repayments α_i. For cash and available-for-sale assets ($i \in \{1, 4, 6\}$), we assume yearly rebalancing, so that $\alpha_i = 1$. For mortgages and personal loans $i \in \{2, 3\}$, we assume the approximation

$\Lambda = 21.5\%$	$\lambda = [100\%\ 0\%\ 0\%\ 100\%\ 100\%\ 50\%\ 50\%]$
$N = 78\%$	$\nu = [0\%\ 65\%\ 85\%\ 5\%\ 5\%\ 5\%\ 5\%]$
$C = 10\%$	$IRR = 1.1\%$
$h = 15\%$	RW=[0% 35% 100% 0% 0% 100% 100%]
$K_1 = K_2 = 110\%$	$S = [100\%\ 0\%\ 0\%\ 100\%\ 100\%\ 100\%\ 100\%]$
$K_3 = 10\%$	$K_4 = 100\%$
$M = 40\%$	

Table 2: Fixed input data for the model

that every year the bank gives a constant amount of new loans and that the amortization is constant. Since the amortization on the loan is a function of the interest rate, for simplification, we posit that all loans have the same initial amount and fixed interest rate corresponding to the average interest rate in the evaluation period (5.46% for mortgages and 11.54% for personal loans). Using this method, we arrive at the amortization rates in Table 3.

For held-to-maturity bonds ($i \in \{5, 7\}$), amortizations occur only at the end of the period. If we make a similar assumption, in that the bank invests in a constant amount every year, let us say c, the bank will have an outstanding amount cM_i in these bonds, on which every year c will amortize and c will be reinvested. The amortization rate is thus $\alpha_i = c/(cM_i) = 1/M_i$. For example, suppose the bank invests 100 million dollars in 10-year Treasuries every year. In that case, the bank will have a constant outstanding amount of 1 billion after ten years, on which 100 million will amortize, corresponding to an amortization rate of 10%, or 1/10.

Additionally, the interest rate on legacy contracts of asset i, $\hat{r}_i$, is initialized as the average rate of the previous 10 years for the first year in our study. Subsequently, $\hat{r}_i$ is updated with $(1 - \alpha_i)\hat{r}_i + \alpha_i r_i$, $\forall i \in A$.

The interest rate on new contracts (r_i) is taken from the end of the previous year. When the bank decides the allocation at the beginning of year t, it knows the interest rates that it will receive on the assets by the end of that year t, given that all assets have fixed rates. This part is deterministic and does not need to be estimated. The bank does not know the default rate on the assets that will occur by the end of year t, so it needs to estimate it. This is

A_i	A_1	A_2	A_3	A_4	A_5	A_6	A_7
M_i	—	30	2	—	10	—	20
α_i	1	0.0518	0.655	1	1/10	1	1/20
LGD_i	0	0.471	0.64	0	0	0.628	0.628

Table 3: Values for the fixed parameters used in the model for each asset.

Parameter	**Value**
ρ_2	0.15
ρ_3	$0.03 \times \dfrac{1 - \mathrm{e}^{-35\overline{PD}_3}}{1 - \mathrm{e}^{-35}} + 0.16 \times \left(1 - \dfrac{1 - \mathrm{e}^{-35\overline{PD}_3}}{1 - \mathrm{e}^{-35}}\right)$
ρ_7	$0.12 \times \dfrac{1 - \mathrm{e}^{-50\overline{PD}_7}}{1 - \mathrm{e}^{-50}} + 0.24 \times \left(1 - \dfrac{1 - \mathrm{e}^{-50\overline{PD}_7}}{1 - \mathrm{e}^{-50}}\right)$

Table 4: Value of parameter ρ_i, $i \in \{2, 3, 7\}$.

estimated by $\overline{PD}_i * LGD_i$, where the probability of default on asset i ($\overline{PD}_i$) was estimated using the (simple) moving average method over the previous 10 years (see [4, 39]). The LGD_i parameters (loss given default) are reported in Table 3 and were obtained from [3, 40] for loans and from [32] for corporate bonds. For mortgages and personal loans ($i \in \{2, 3\}$), we don't have statistics of historical default rates, so the expected loss $\overline{PD}_i * LGD_i$ was estimated using the ten-year moving average of the the previous charge-off rates [16, 17].

We would like to emphasize that the prospective return r_i on AFS bonds is given by y_i, where y_i is the yield on AFS bonds at the end of the previous year.

Table 4 reports the ρ_i values, $i \in \{2, 3, 7\}$ that correspond to the correlation between different contracts within the same asset class. These were used to compute the risk of assets $i \in \{2, 3, 7\}$ (given by the credit VaR) and were taken from [1, 5, 12].

3 Computational experiments

In this section, we report an extensive computational study comparing the proposed model with classical heuristic strategies, and we evaluate the *ex-post* performance, using historical data.

Since the heuristic approaches may not give a solution that is compliant under Basel III, we search the nearest solution that verifies these constraints by solving the following model:

$$\begin{aligned} \min_{x\in\Omega} \quad & ||x - x^H|| \\ \text{subject to} \quad & (2) - (12), \end{aligned} \tag{18}$$

where x^H is a heuristic solution. In this research, we consider norm ℓ_1, that is, $||x - x^H|| = \sum_{i\in A} |x_i - x_i^H|$, since it allows us to get solutions that modify fewer components of the original one, making this approach less sensitive to parameters than other norms, such as the Euclidean norm. However, other norms could be considered.

3.1 Tested approaches

Altogether, six approaches (three optimized and three heuristics) were tested in this work. Three of them come from the optimized model suppressing some turnover constraints to better understand their effect in the final solution. These strategies are:

- M1: this approach consists of applying the original model presented in section 2;
- M2: similar to the previous one but removing upper bound local turnover constraint (10);
- M3: similar to the previous one but removing also the global turnover constraint (9).

We compare the optimized approaches against classical heuristic approaches, which we list below:

- EW (Equal Weighting): all the assets have the same allocation in the balance sheet;

$$x^{EW} = \left[\frac{100}{7}\% \ \frac{100}{7}\% \ \frac{100}{7}\% \ \frac{100}{7}\% \ \frac{100}{7}\% \ \frac{100}{7}\% \ \frac{100}{7}\%\right].$$

- 60/40: this is an adaptation of the 60/40 equity/bond portfolio allocation [9] to banks. In this work, we use this strategy to define a balance sheet allocating 60% in assets with high risk and 40% to assets with lower risk. We consider the cut-off point for risk as 2% and set equal weightings inside each group. Taking into account the average risk reported in Table 7, this leads to the following allocation:

$$x^{60/40} = \left[\frac{40}{3}\% \ \frac{60}{4}\% \ \frac{60}{4}\% \ \frac{60}{4}\% \ \frac{40}{3}\% \ \frac{60}{4}\% \ \frac{40}{3}\%\right].$$

- RP (Risk Parity): this strategy makes the allocations in such a way that all the assets contribute with the same risk to the final solution. Then, in case $\sigma_i > 0$, $\forall i \in A$, this solution could be defined as $x_i = \frac{1/\sigma_i}{\sum_{j \in A} 1/\sigma_j}$, $i \in A$. However, some assets have no risk or very low risk, so that we need to adapt the methodology to our bank setting. Consequently, for a specific year of the simulation, we define the set of assets A_R that have a risk penalty greater than 2% and apply a risk parity strategy to these assets, and an equal weighting strategy to the remaining ones. This set has to be updated for each year in the simulation. To compare the behaviour of this solution with the previous one, we keep the same allocation proportion between high-risk and low-risk assets. Thus, this solution is given by

$$x_i^{RP} = 0.6 \times \frac{1/\sigma_i}{\sum_{j \in A_R} 1/\sigma_j}, \quad i \in A_R, \tag{19}$$

and

$$x_i^{RP} = \frac{0.4}{\#(A \backslash A_R)}, \quad i \notin A_R. \tag{20}$$

3.2 Initial balance sheet

In order to assess the robustness of the results on the performance of the optimized and heuristic strategies, we defined seven different initial balance sheets (see Table 5), which are defined as follows:

A: allocate 50% of the balance sheet to *Cash*;

B: allocate 50% of the balance sheet to *Loans*;

C: distribute the asset allocation evenly;

D: typical asset structure of a diversified *retail bank*. We consider a retail bank that deals mostly with individuals, so it focuses on solutions for these costumers, such as personal loans and mortgages. The bank should have enough liquid securities and cash to withstand liquidity shocks, thus showing a significant allocation to liquid assets;

E: typical asset structure of an *investment bank*. Investment banks usually deal with securities. Therefore, the example we consider reflects the activity in investment securities;

F: typical asset structure of a *consumer credit bank*. The consumer credit bank we consider has its core activity in consumer credit or personal loans, but will also have to allocate a significant amount to liquid assets such as Treasury bonds or cash to withstand unforeseen liquidity shocks;

G: typical asset structure of a *mortgage loan bank*. In this case, we consider a bank whose core activity is granting mortgage loans. Thus, its balance sheet reflects its activity. Naturally, the bank will still have to allocate a significant amount to liquid assets.

All of these initial balance sheets are compliant under Basel III.

	Initial balance sheet						
Assets	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Cash	50%	10%	$\frac{100}{7}$%	5%	20%	25%	10%
Mortgage loans	10%	20%	$\frac{100}{7}$%	40%	0%	0%	60%
Personal Loans	10%	30%	$\frac{100}{7}$%	20%	0%	50%	0%
Treasury bonds AFS	7.5%	15%	$\frac{100}{7}$%	25%	40%	0%	10%
Treasury bonds HTM	7.5%	10%	$\frac{100}{7}$%	5%	20%	25%	10%
Corporate bonds AFS	7.5%	10%	$\frac{100}{7}$%	2.5%	20%	0%	5%
Corporate bonds HTM	7.5%	5%	$\frac{100}{7}$%	2.5%	0%	0%	5%

Table 5: Description of the seven initial balance sheets (corresponding to the balance sheet for 1994 in our simulation).

3.3 Out-of-sample results

We use historical data used to evaluate the performance of the proposed model, namely public USA data for interest rates and defaults from 1985 until 2022. Table 6 indicates the data sources, and Table 7 reports a summary overview of the average return and the average risk penalty for each asset.

Asset structure	**Interest rate**	**Defaults**
Cash	[18]	—
Mortgage loans	[15]	[17, 40]
Personal loans	[19]	[16, 3]
Treasury bonds AFS	[14]	—
Treasury bonds HTM	[14]	—
Corporate bonds AFS	[20]	[32, Exhibit 23 (page 28) and Exhibit 7 (page 8)]
Corporate bonds HTM	[20]	[32, Exhibit 23 (page 28) and Exhibit 7 (page 8)]

Table 6: Sources for the rates for each asset.

Although we have available data from 1985 to 2022, in order to obtain a solution in the *ex-ante* optimization process, we need to predict some parameters of our model (PD_i, r_i, σ_i), smoothing their values with the average over

Asset structure	**Return** (%)	**Risk penalty** (%)
Cash	2.402	0.000
Mortgage loans	5.176	4.679
Personal Loans	9.067	7.370
Treasury bonds AFS	5.264	8.726
Treasury bonds HTM	3,870	0.000
Corporate bonds AFS	7.609	7.178
Corporate bonds HTM	6.278	1.348

Table 7: Average return and risk penalty for each asset during the evaluation period (1995 − 2022).

the previous 10 years, as discussed in section 2.2. Consequently, our *ex-post* simulation only runs from 1995 to 2022.

At year t, the *ex-post* simulation consists on evaluating the solution obtained in the *ex-ante* optimization process with the effective return function at the end of that year:

$$\tilde{r}(x) = x_1 r_1 + \sum_{i \in A_L} (\hat{x}_i \hat{r}_i + \tilde{x}_i r_i - x_i LGD_i PD_i) + \sum_{i=4,6} x_i \tilde{r}_i, \tag{21}$$

where r_i is the actual value of the interest rate at the end of the previous year $t-1$, for $i = 1$ or $i \in A_L$, and PD_i is the default rate observed at the end of year t. Since all assets have fixed interest rates, the interest rates that the bank receives are from the end of $t-1$, knowing that the bank decided the allocation at the beginning of year t. When the bank decides the allocation at the beginning of year t, it does not know the default rate that will occur at the end of that year. This will be only known by the end of the year t.

For AFS bonds (i=4,6) and year t, the effective return r_i is given by

$$\tilde{r}_i = r_i - Dur(r_i)\Delta, \tag{22}$$

where r_i is the yield on AFS bonds at the end of the previous year $t-1$, $Dur(y)$ is the modified duration on the bonds, and Δ is the increase/decrease in interest rates from the beginning to the end of the year. Let us first give the intuition for Equation (22) and then explain in detail how it is computed. Recall that, unlike held-to-maturity bonds, available-for-sale bonds are registered at market prices in the balance sheet, so their price variations affect the return on the bank. The effective return is thus governed by two parts. The first is the yield on the bonds, which is a proxy for the income that the bonds generate. The second is the variation in market prices which can be approximated by multiplying the negative of the modified duration on the bonds by the interest rate variation. Let us give an example. Suppose that a 10-year Treasury bond has a modified duration of 9, that in the beginning of the year t (end of year $t-1$) has a yield of 4%, and that at the end of the year t the yields have increased from 4% to 5%. The effective return can be approximated by two parts: the first is the income, which is given by 4%, and the second is the variation in market prices, which can be approximated by the duration times the variation in yields, which is equal to $-9 * 1\% = -9\%$. As a result, the effective return for year t in this example is $4\% - 9\% = -5\%$.

For the sake of clarity, we review the computation of the modified duration for the bond. Let us denote $P(y)$ the bond price as a function of its yield y, with a coupon c and maturity T. Assuming yearly coupon payments, the bond price can be determined by

$$P(y) = \frac{1}{(1+y)^T} + \sum_{t=1}^{T} \frac{c}{(1+y)^t}. \tag{23}$$

where the first term is the present value of the principal payment and the second corresponds to the annuity associated with coupon payments. The previous expression can be cast as

$$P(y) = \frac{1}{(1+y)^T} + \sum_{t=1}^{T} \frac{c}{(1+y)^t} = \frac{1}{(1+y)^T} + \frac{c}{(1+y)} \sum_{t=0}^{T-1} \frac{1}{(1+y)^t} \tag{24}$$

$$= \frac{1}{(1+y)^T} + \frac{c}{(1+y)} \frac{1 - \left(\frac{1}{1+y}\right)^T}{1 - \left(\frac{1}{1+y}\right)} \tag{25}$$

$$= \frac{1}{(1+y)^T} + \frac{c}{y}\left[1 - \left(\frac{1}{1+y}\right)^T\right] \tag{26}$$

$$= \frac{c}{y} + \frac{1}{(1+y)^T}\left[1 - \frac{c}{y}\right]. \tag{27}$$

The modified duration $Dur(y)$ is a measure of the interest rate risk on the bond, which allows one to estimate the impact of an increase or decrease in interest rates, as we have exemplified above. Bonds with longer durations tend to be riskier. The modified duration is given by

$$Dur(y) = -\frac{dP(y)/dy}{P(y)}. \tag{28}$$

In our setting we assume that AFS bonds are rebalanced every year, and that the new investments in bonds are made at par, which means that $c = y$ and $P(y) = 1$. The modified duration therefore is given by first differentiating expression (27) and then setting $c = y$, yielding

$$Dur(y) = \frac{1}{y} - \frac{1}{y(1+y)^T}. \tag{29}$$

Figure 2 shows the evolution of the accumulated effective return, r^a, for each one of the strategies tested and the initial balance sheets considered. It is computed as

$$r_0^a = 100 \quad \text{and} \quad r_t^a = r_{t-1}^a(1 + \tilde{r}(x_t)), \quad t > 0, \tag{30}$$

where t is the index of each one of the years under study and $x_t = (x_{t,1}, \ldots, x_{t,7})$ is the corresponding solution computed in the beginning of that year. Notice that the models presented (original model presented in section 2 and model described in section 3) are one-period models that are executed consecutively over the years so that the solution in a given year is determined at the expense of the previous one.

The flowchart (Figure 1) shows the simulation procedure used. Algorithms are presented below in pseudocode to make the concept clearer after a brief explanation.

Algorithm 1 allows computing the solution (x_t) to be implemented in the beginning of year t and depends on the chosen model and the legacy balance sheet received from the previous year. The solution obtained in this algorithm is then used in Algorithm 2 that, after the initialization step (step 2) and for each year in the time windows considered (step 3), updates the balance sheet to be executed in the beginning of year t (step 4); this algorithm subsequently computes the effective return at the end of year t (step 5) and the accumulated effective return at the end of year t (step 6). Finally, Algorithm 3 gives us a sketch of the simulation procedure, where we compared the 3 mathematical models (M_1, M_2, M_3) described in Section 2 and the 3 heuristics ($EW, 40/60, RP$) considered in Section 4.

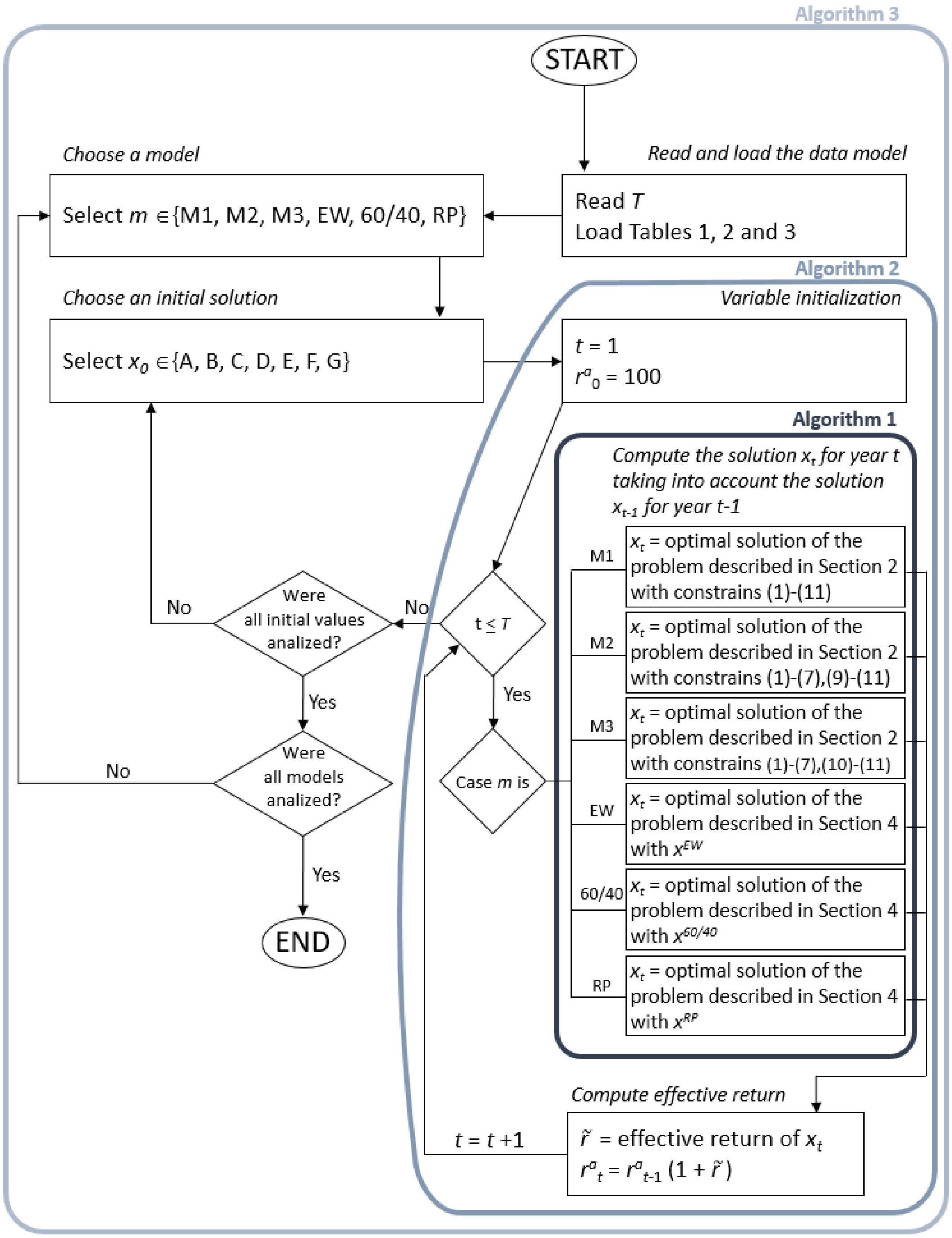

Figure 1: Simulation procedure flowchart

Algorithm 1 Algorithm to determine the solution for a new year given a model and the solution of the previous year.

```
procedure x* = SOLUTIONAPPROACH(x^0, model, r, r̂, PD̄, σ)
    if model ∈ {M_1} then
        return x* = optimal solution of the problem described
                    in Section 2 with constrains (1)-(11);
    if model ∈ {M_2} then
        return x* = optimal solution of the problem described
                    in Section 2 with constrains (1)-(7), (9)-(11);
    if model ∈ {M_3} then
        return x* = optimal solution of the problem described
                    in Section 2 with constrains (1)-(7), (10)-(11);
    if model ∈ {EW} then
        return x* = optimal solution of the problem described
                    in Section 4 with x^EW;
    if model ∈ {40/60} then
        return x* = optimal solution of the problem described
                    in Section 4 with x^{40/60};
    if model ∈ {RP} then
        return x* = optimal solution of the problem described
                    in Section 4 with x^RP;
```

Algorithm 2 Computation of the accumulated effective return.

procedure ACUMEFFECTIVERETURN(x^0, *model*, T)
  ▷ Initialization
  $r_0^a \leftarrow 100$;
  $\hat{r} \leftarrow \frac{1}{10}\sum_{j=1}^{10} r_{1-j}$;
  $\sigma_1 \leftarrow 0$; $\sigma_5 \leftarrow 0$;
  ▷ Solution path
  for $t = 1$ to T **do**
    $\overline{PD} \leftarrow \frac{1}{10}\sum_{j=1}^{10} PD_{t-j}$; ▷ Estimated PD_t
  ▷ Estimation of penalty risk
    $\sigma_i \leftarrow UL_i(0.999) - EL_i, i \in \{2,3,7\}$; ▷ Credit VaR
    $\bar{r} \leftarrow \frac{1}{10}\sum_{j=1}^{10} r_{t-j}$;
    $\sigma_i \leftarrow N^{-1}(0.95)\sqrt{\frac{1}{10}\sum_{j=1}^{10}(r_{t-j} - \bar{r})^2}, i \in \{4,6\}$; ▷ Market VaR
    $x_t \leftarrow$ solutionApproach(x_{t-1}, *model*, r_{t-1}, $\hat{r}$, $\overline{PD}$, σ);
    $\hat{r} \leftarrow (1-\alpha)\hat{r} + \alpha r_t$; ▷ Update of legacy returns
  ▷ Compute effective return
    $\tilde{r}_i \leftarrow r_{t,i} - PD_{t,i} * LGD_i, i \in \{1,2,3,5,7\}$;
    $\tilde{r}_i \leftarrow r_{t-1,i} - (\frac{1}{r_{t-1,i}} - \frac{1}{r_{t-1,i}(1+r_{t-1,i})^{M_i}})(r_{t,i} - r_{t-1,i}), i \in \{4,6\}$;
    $\tilde{r} \leftarrow \tilde{r}_t^\top x_t$; ▷ effective return of solution x_t
    $r_t^a \leftarrow (1+\tilde{r}) r_{t-1}^a$; ▷ accumulated effective return in year t
  return r_T^a

Algorithm 3 Simulator procedure.

procedure SIMULATION(T)
  Load Tables 1, 2 and 3;
  Read T
  for $model \in \{M_1, M_2, M_3, EW, 40/60, RP\}$ **do**
    for $x^0 \in \{A,B,C,D,E,F,G\}$ **do**
      $r_{model,x^0}^a \leftarrow$ acumEffectiveReturn(x^0, *model*, T);

The evolution of the allocation for these solutions is presented in Figures 3 - 9 (see the annex section). Table 8 summarizes the accumulated effective

Strategy	Initial balance sheet A	B	C	D	E	F	G
M_1	693.338	760.171	773.512	764.127	526.912	728.770	691.136
M_2	744.107	797.012	799.521	815.475	765.217	774.616	820.094
M_3	813.458	820.666	822.837	845.664	801.622	803.806	845.956
EW	546.830	573.969	573.848	570.998	376.969	537.538	540.952
60/40	554.267	581.636	581.309	578.035	377.038	537.283	545.577
RP	513.692	572.313	570.460	562.435	385.640	517.123	533.804
M_4	789.344	785.791	793.872	789.948	771.917	771.682	777.700

Table 8: Accumulated effective return in the last year of the simulation.

Strategy	Initial balance sheet A	B	C	D	E	F	G	Average
Optimized	7.74	7.97	8.00	8.04	7.40	7.85	7.92	7.84
Heuristic	6.43	6.70	6.69	6.66	5.07	6.38	6.45	6.34
Difference	1.31	1.27	1.30	1.38	2.33	1.47	1.47	1.51

Table 9: Comparison between the average accumulated effective return on both types of strategy (values in percentage).

return in the last year of the *ex-post* simulation and Table 9 compares the average results of optimized approaches with the heuristic ones.

These results attested the greater performance of the optimized solutions over heuristics in the ex-post simulation, reaching an average annual outperformance of 1.51% in terms of return on assets and 15.1% in terms of return on equity, since we consider a capital allocation of 10%.

When the constraints are removed from the model presented in section 2, the performance increases but the stability is penalized. For example, the strategy $M3$, which does not use turnover constraints, shows annual variations in the allocations above 40%, which does not happen in practice, as the allocations in the banking sector tend to be rigid for the reasons explained in the introduction.

The heuristic approaches show similar out-of-sample performances at the end of the simulation. Finally, as the initial balance sheets are different from the heuristic solutions, we can observe that the allocations converge to the

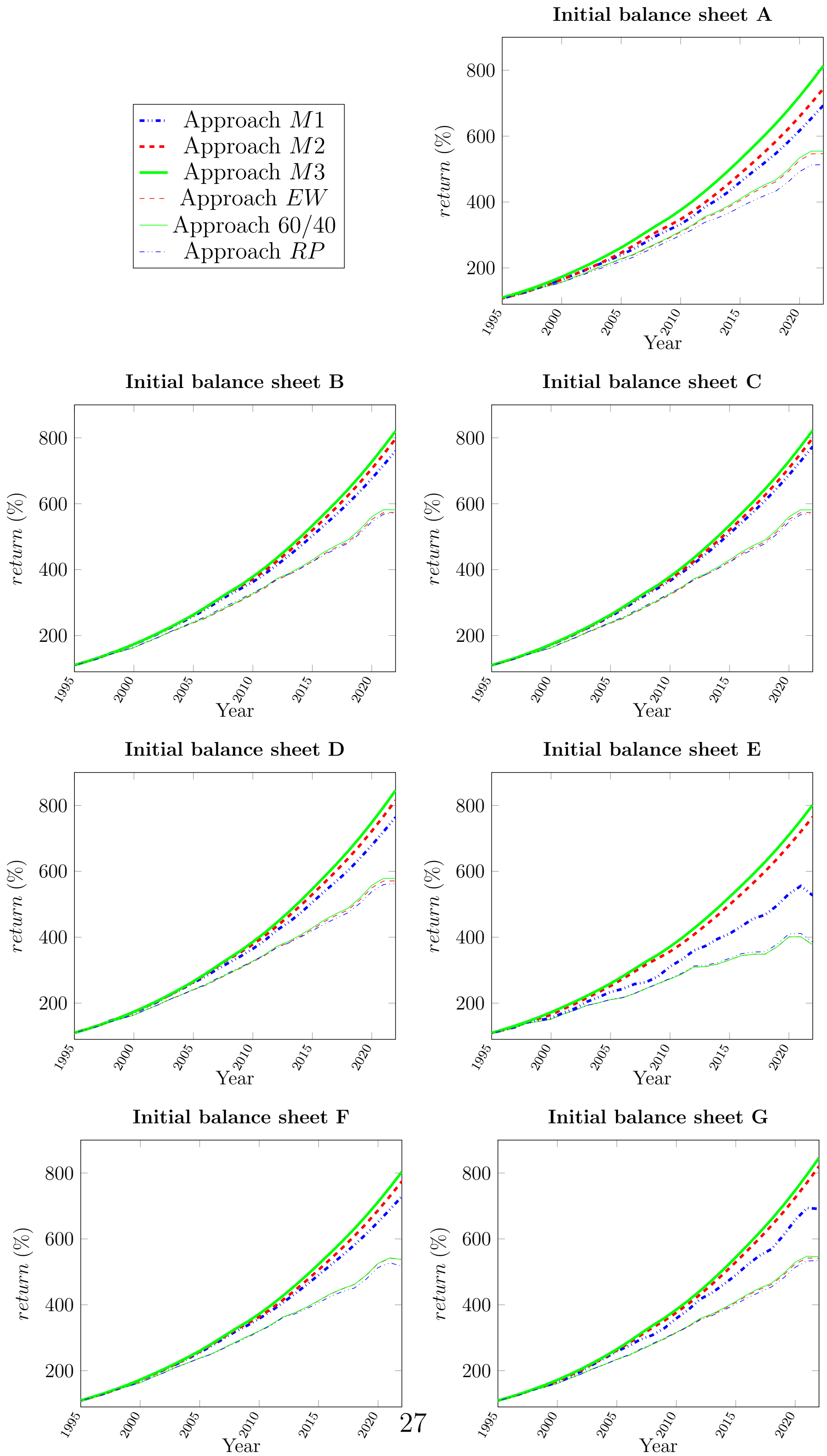

Figure 2: Accumulated effective return from 1995 to 2022.

heuristic solution.

To assess the effect of the constraints in our model, we consider version M_4 allowing sales of legacy loans without limits on turnover constraints by penalizing the sales on legacy loans with transaction costs of 20%. Transaction costs of such magnitudes for loan portfolios are reported for example in [31]. The results in Table 8 show that, in general, models M_2 and M_3 outperform this strategy. However, model M_4 outperforms strategy M_1 overall, which reinforces the idea that excessive turnover restrictions have a negative impact on the efficiency of the method.

4 Conclusion

In this paper, we developed a testing framework for bank portfolio optimization by comparing a properly calibrated model with turnover constraints to heuristic strategies. For that purpose, we used an extensive historical data set to evaluate each strategy's out-of-sample performance and stability.

Our testing framework allowed us to confirm a series of conclusions:

1. When we remove global turnover constraints from the optimization models, this results in excessive variations in the allocations that cannot be implemented in practice.

2. Optimization models with turnover constraints show smoother allocations and therefore can be implemented in an industrial context.

3. Optimization models show superior out-of-sample performance when compared to heuristic strategies. Notice that this is not a given, as numerous studies have shown that, in the case of portfolio optimization, often heuristic strategies outperform portfolio optimization strategies when considered out-of-sample. Using our dataset, we report an increase in annual ex-post outperformance of 1.51% in terms return on assets and of 15.1% in terms return on equity.

These findings confirm that properly calibrated bank optimization models can be used in practice and outperform considerably heuristic strategies, in contrast to many of the investment portfolio optimization models which have been shown to underperform, as highlighted for instance in [11]. Thus, this research enables the use of bank optimization methodologies by banks in practice.

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Annex

In this annex, we develop further tests on the evolution of the asset allocations for the different methodologies highlighted in the paper (M_1, M_2, M_3, $EW, 60/40, RP$). In order to assess the robustness of the results, we use different initial balance sheets, which are highlighted in Table 5. The analysis of the results is conducted in Section 3.3. These tests are critical to assess the stability of the different balance sheet approaches. For further details, please refer to Section 3.3.

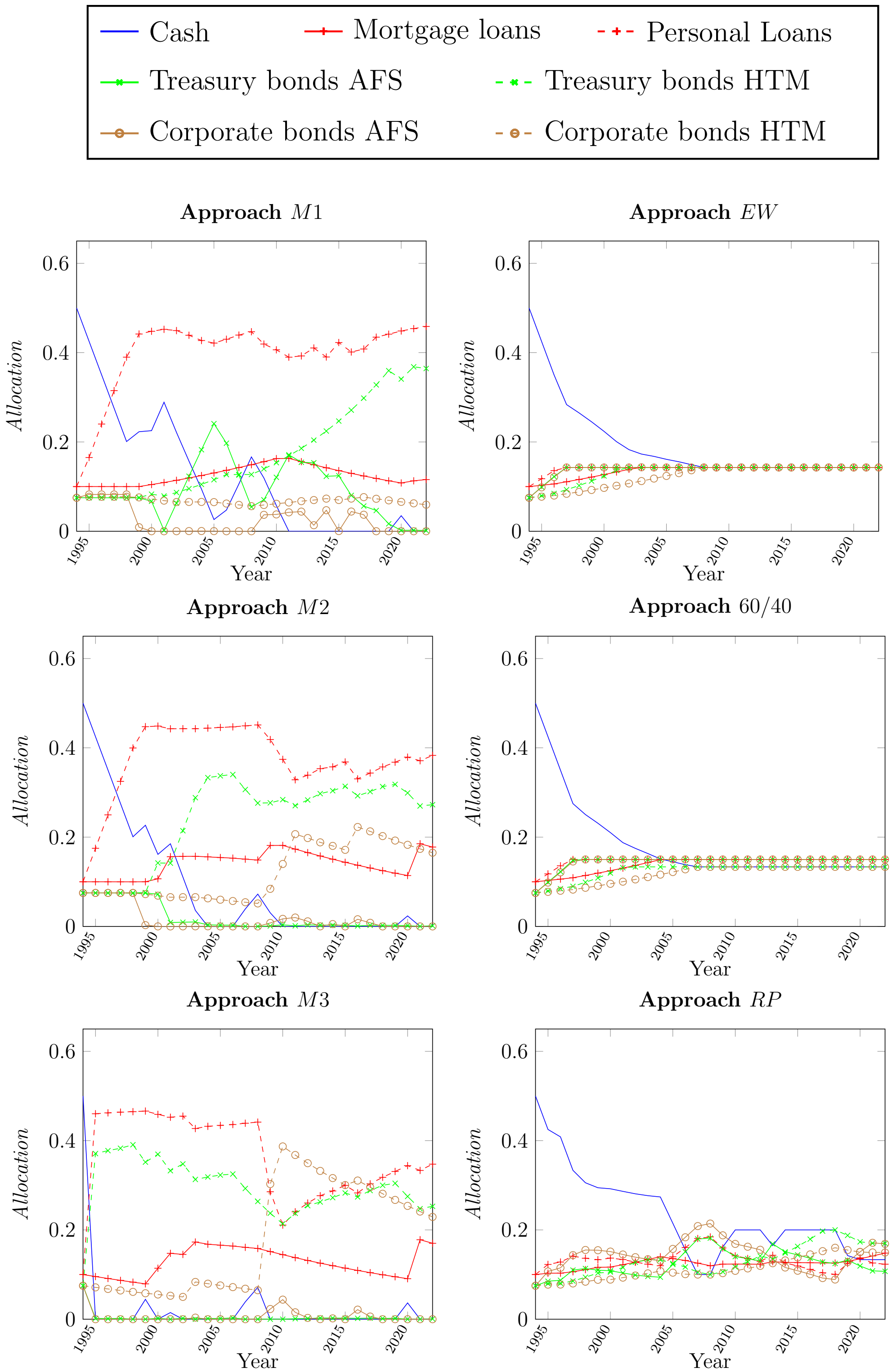

Figure 3: Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2022 (initial balance sheet A).

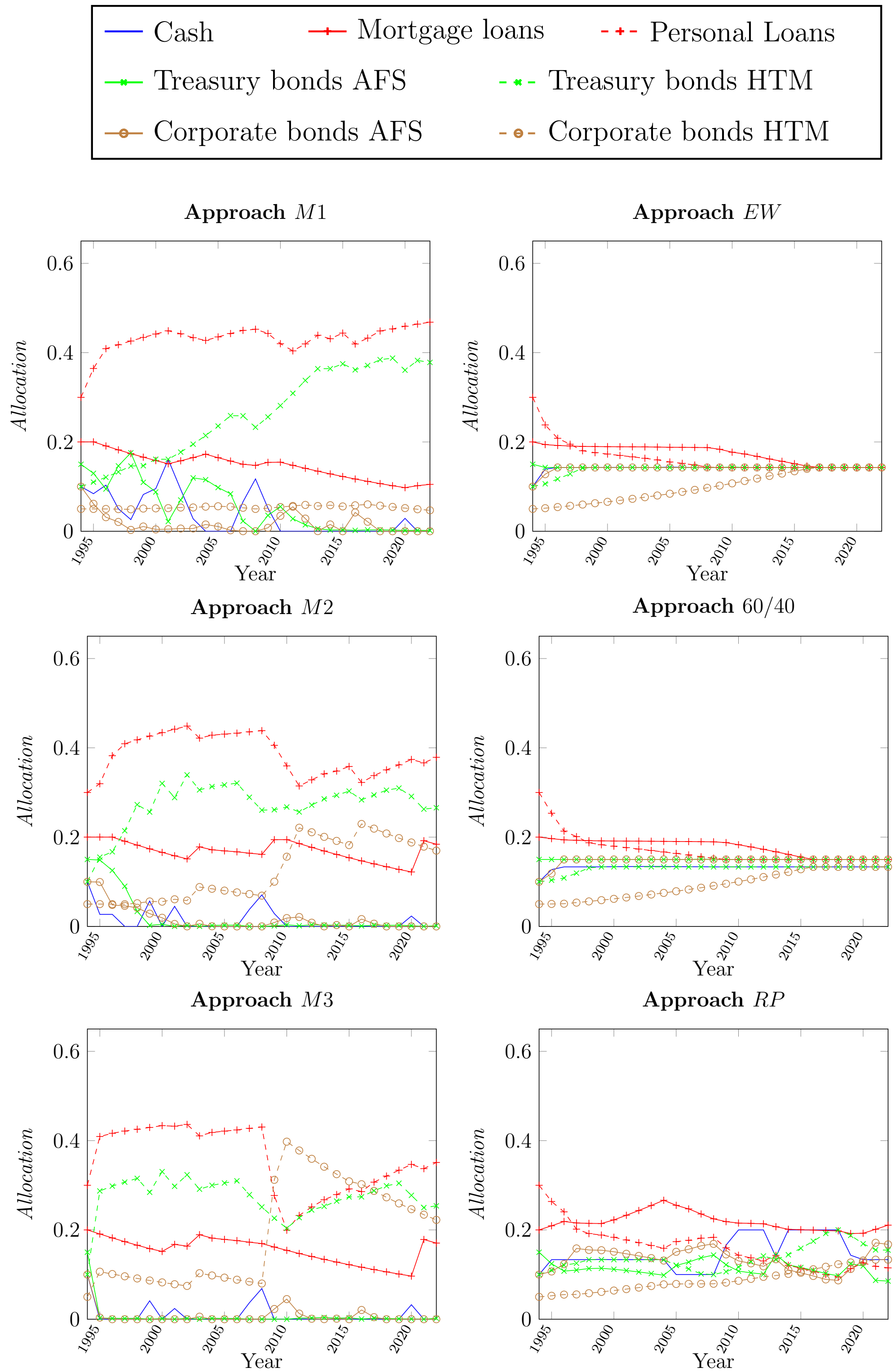

Figure 4: Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2022 (initial balance sheet B).

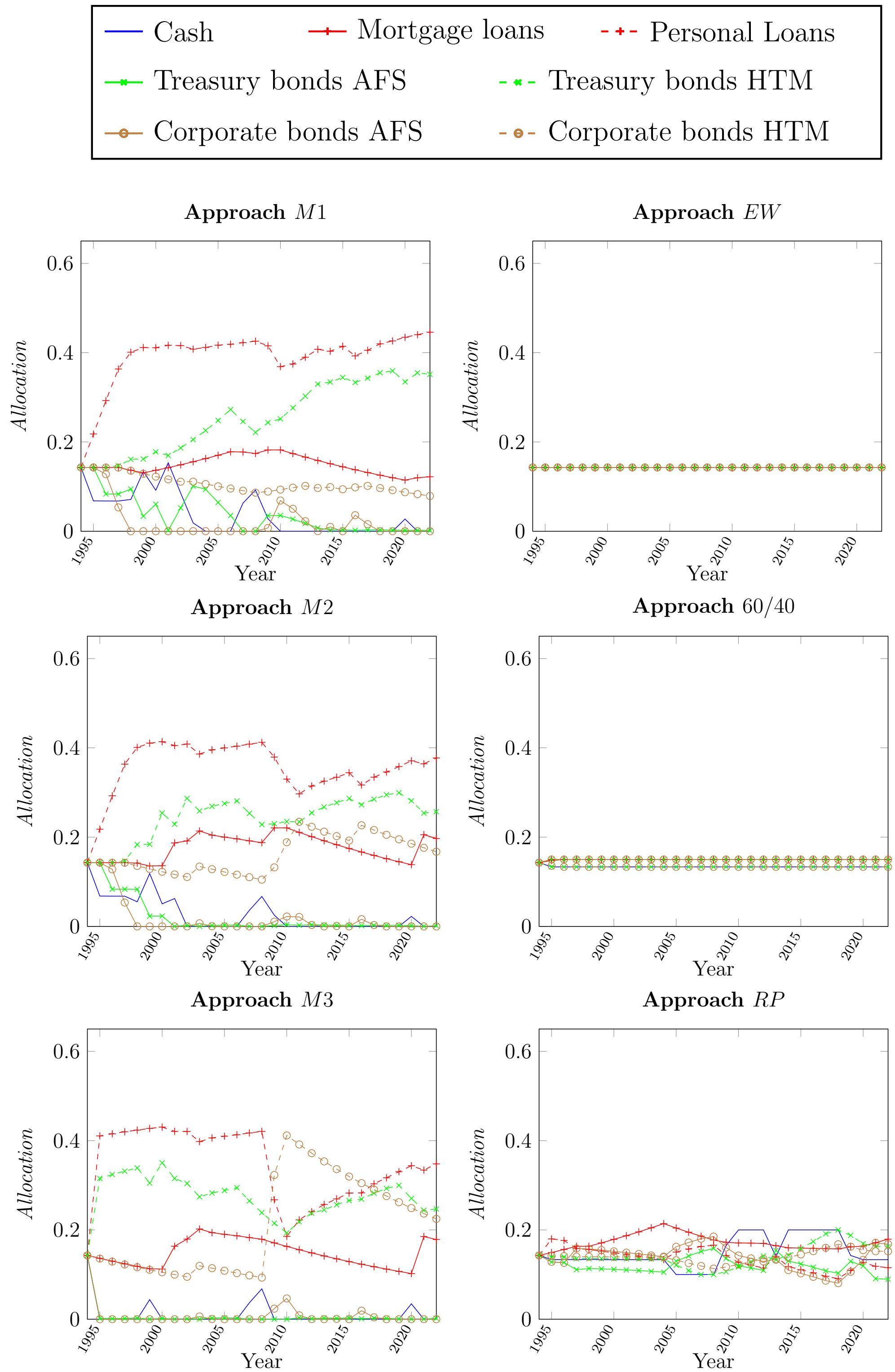

Figure 5: Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2022 (initial balance sheet C).

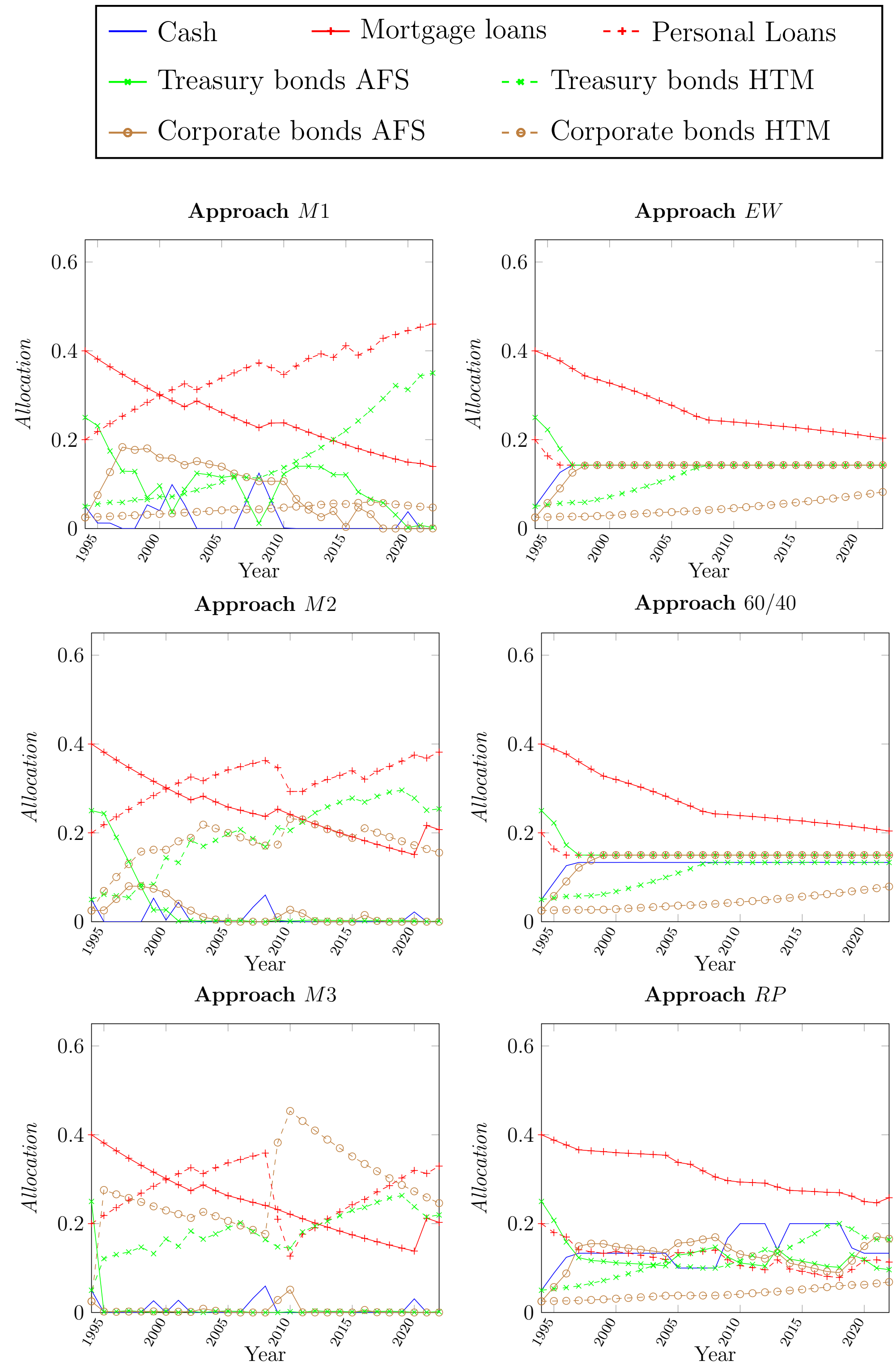

Figure 6: Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2022 (initial balance sheet D).

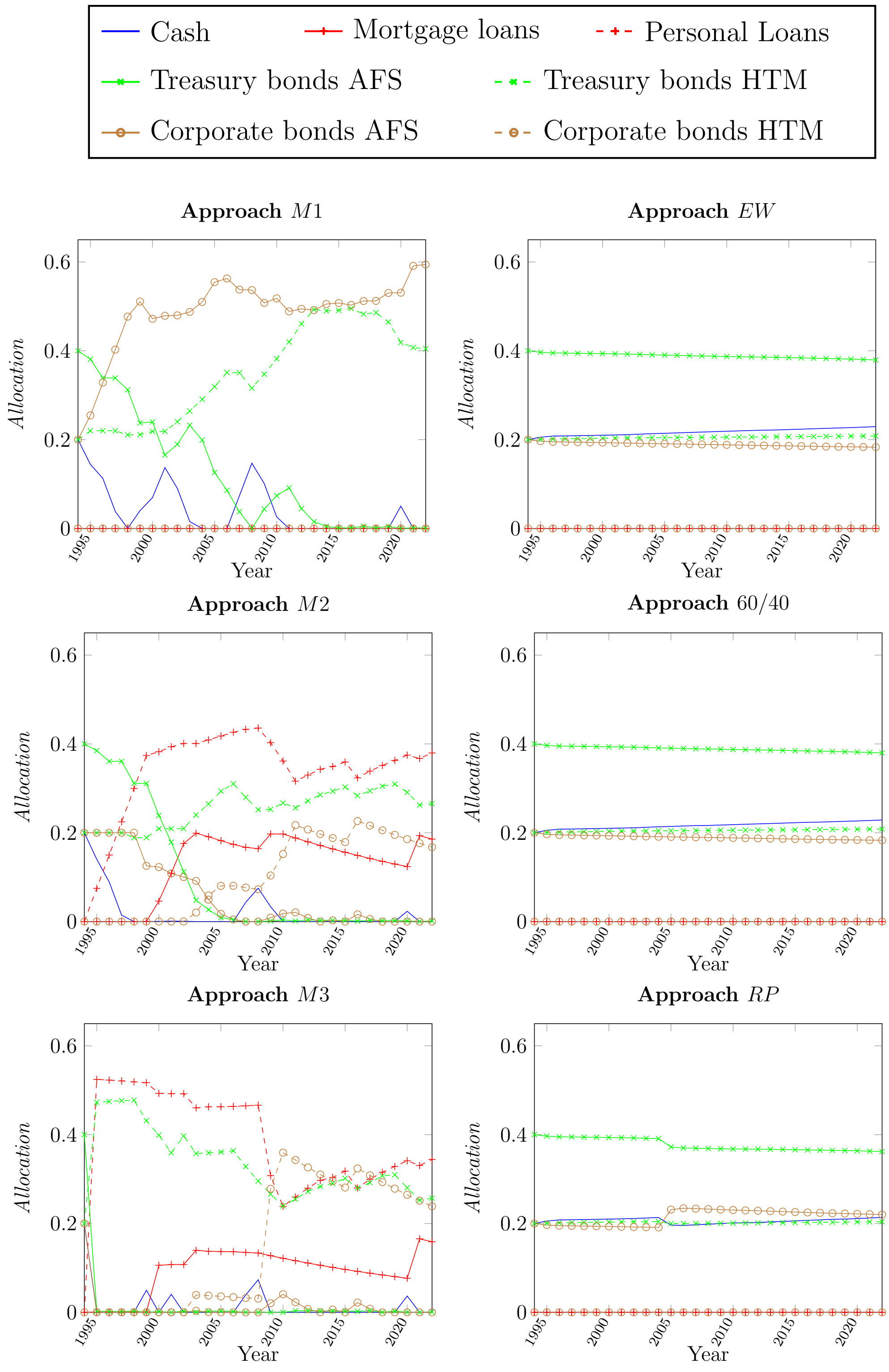

Figure 7: Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2022 (initial balance sheet E).

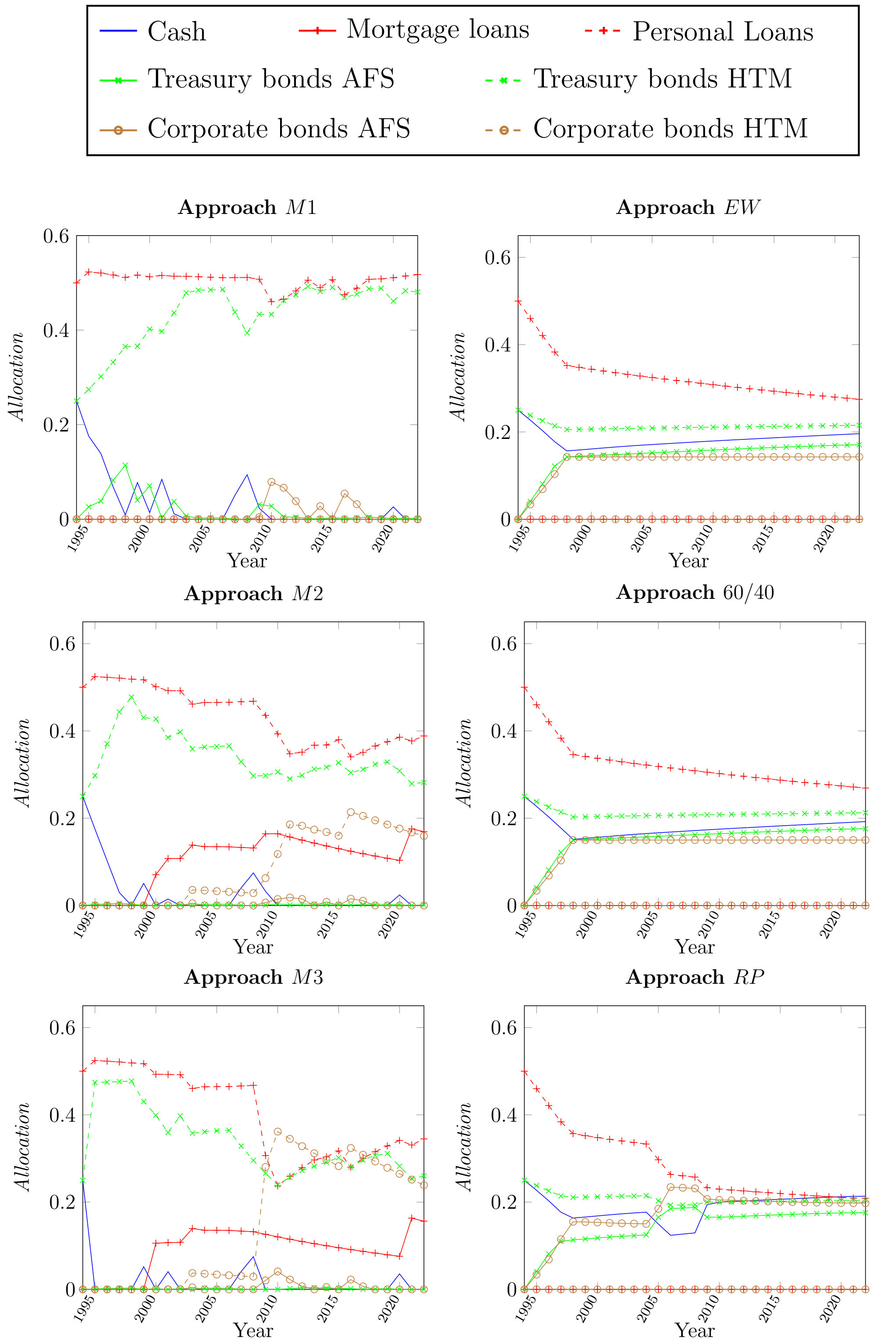

Figure 8: Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2022 (initial balance sheet F).

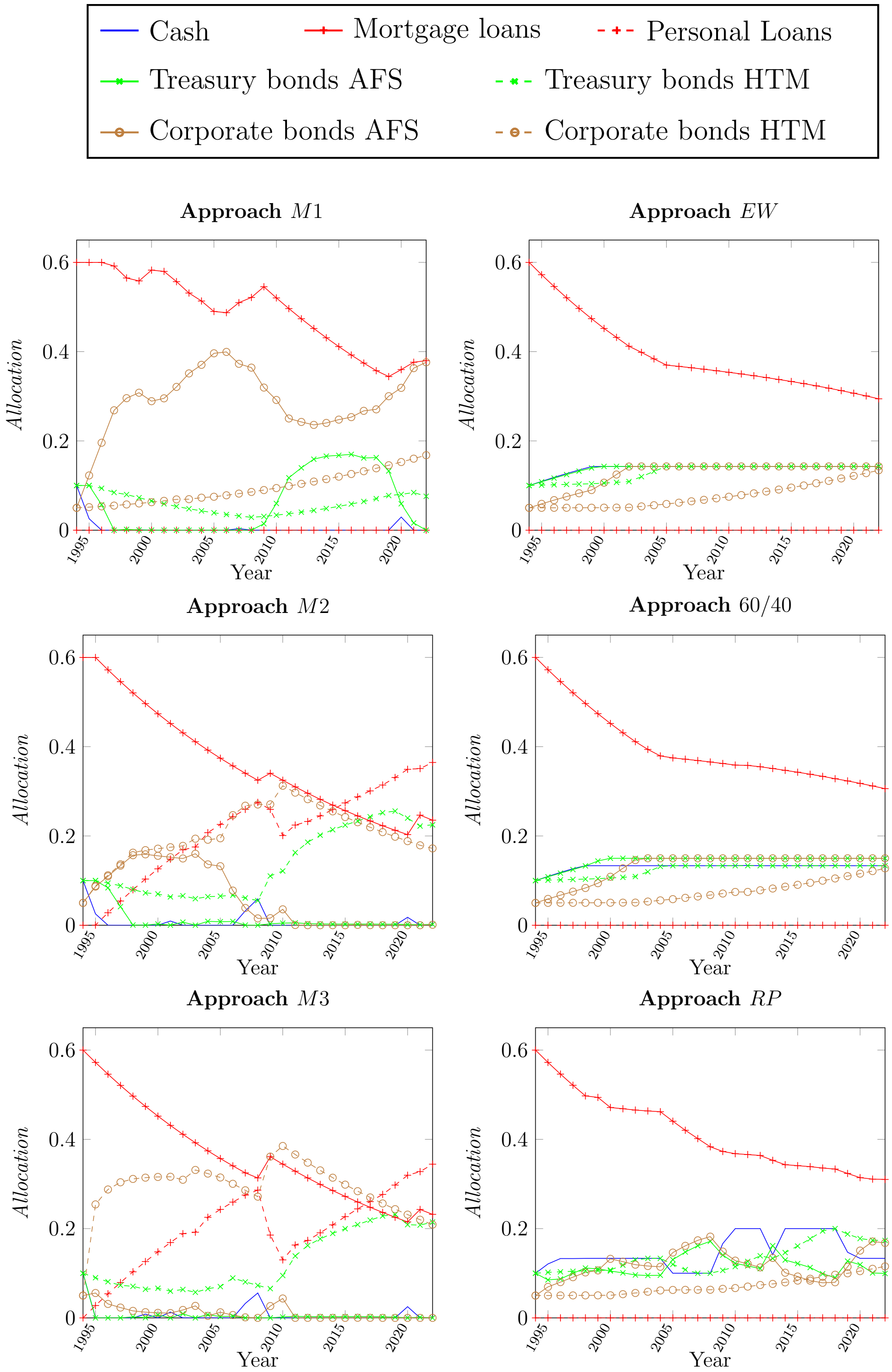

Figure 9: Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2022 (initial balance sheet G).